PET ENGINEERING COLLEGE

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

UNIT - IV

CLASS	**: S5 ECE**
SUBJECT CODE	**: EC3551**
SUBJECT NAME	**: TRANSMISSION LINES AND RF SYSTEMS**
REGULATION	**: 2021**

Unit - 4

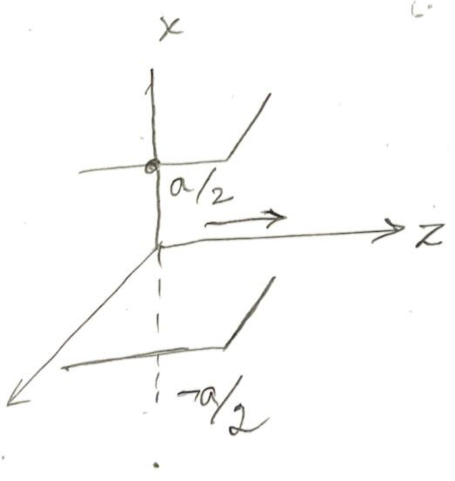
Waveguides

Guided waves:

When the fields are confined by boundaries of material different from transmission path. This wave is called guided wave

Electromagnetic wave between parallel plane,

Consider an electromagnetic wave propagating between perfectly two parallel Conductors. Conductors are placed at a distance of $a/2$ and $-a/2$



$$\nabla \times H = \sigma E + \frac{dD}{dt}$$

$$\nabla \times E = -\frac{dB}{dt}$$

$$\nabla \cdot D = \rho$$

$\nabla \cdot B = 0$

When the parallel plate is placed at a distance in x axis, the waves move in z direction. The electric & magnetic field is constant in y direction.

For electric field:

The field is constant in Y direction.

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_z}{\partial y} = \frac{\partial H_x}{\partial y} = \frac{\partial H_z}{\partial y} = 0$$

$$\nabla \times E = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix}$$

$$\nabla \times E = a_x\left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right] - a_y\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] + a_z\left[\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right]$$

$$\nabla \times E = a_x\left(\frac{-\partial E_y}{\partial z}\right) - a_y\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] + a_z\left[\frac{\partial E_y}{\partial x}\right] \rightarrow (1)$$

$$\nabla \times E = -\frac{dB}{dt} = -\mu \frac{dH}{dt}$$

$$\nabla \times E = -\mu j\omega H$$

$$= -\mu j\omega [a_x H_x + a_y H_y + a_z H_z]$$

$$\nabla \times E = -\mu j\omega a_x H_x - \mu j\omega a_y H_y - \mu j\omega a_z H_z$$

$\rightarrow (2)$

$(1) = (2)$

$$-a_x \frac{\partial E_y}{\partial z} = -\mu j\omega a_x H_x$$

$$-\frac{\partial E_y}{\partial z} = -\mu j\omega H_z \qquad -\partial/\partial z = \gamma$$

$$\gamma E_y = -\mu j\omega H_z \quad — A_1$$

$$-a_y \left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] = -\mu j\omega a_y H_y$$

$$-a_y \frac{\partial E_z}{\partial x} + a_y \frac{\partial E_x}{\partial z} = -\mu j\omega a_y H_y$$

$$\cancel{-}\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] = \cancel{-}\mu j\omega H_y$$

$$\frac{\partial E_z}{\partial x} - \gamma E_x = \mu_0 j\omega H_y \rightarrow A2$$

$$a_z \left[\frac{\partial E_y}{\partial x} \right] = -\mu_0 j\omega a_z H_z$$

$$\frac{\partial E_y}{\partial x} = -\mu_0 j\omega H_z. \Rightarrow A3$$

2. For H field:

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_z}{\partial y} = \frac{\partial H_x}{\partial y} = \frac{\partial H_z}{\partial y} = 0$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$\nabla \times H = a_x \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] - a_y \left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right] + a_z \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right]$$

$$\nabla \times H = -a_x \frac{\partial H_y}{\partial z} - a_y \left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right]$$

$$+ a_z \left[\frac{\partial H_y}{} \right.$$

$$\nabla \times H = \sigma E + \frac{d \varepsilon E}{dt}$$

$B = \mu H$

$D = \varepsilon E$

$\frac{d}{dt} = j\omega$

$$= \sigma E + j\omega \varepsilon E$$

$$= E(\sigma + j\omega \varepsilon)$$

$$\nabla \times H = (\sigma + j\omega\varepsilon)[a_x E_x + a_y \cancel{E_y} + a_z \cancel{E_z}]$$

$\rightarrow (2)$

$(1) = (2)$

$$-a_x \frac{\partial H_y}{\partial z} = (\sigma + j\omega\varepsilon) a_x E_x$$

$$-\frac{\partial H_y}{\partial z} = (\sigma + j\omega\varepsilon) E_x$$

$$+\gamma H_y = (\sigma + j\omega\varepsilon) E_x \rightarrow B_1$$

$$-a_y \left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right] = (\sigma + j\omega\varepsilon) a_y \cancel{E_y}$$

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -(\sigma + j\omega\varepsilon) E_y.$$

$\rightarrow B_2$

$$a_x \left(\frac{\partial H_y}{\partial x}\right) = (\sigma + j\omega\varepsilon) a_z \cancel{E_z}$$

$$\frac{\partial H_y}{\partial x} = (\sigma + j\omega\varepsilon) E_z \rightarrow B_3$$

10/23

(i) For TM wave

E_x, E_z, H_y exists

$H_x, H_z, E_y = 0$

$A_1 \Rightarrow 0$

$$A_2 \Rightarrow \frac{\partial E_z}{\partial x} + \gamma E_x = \mu j\omega H_y \rightarrow (1)$$

$A_3 \Rightarrow 0$

$$B_1 \Rightarrow \gamma H_y = (\sigma + j\omega\varepsilon) E_x \rightarrow (2)$$

$B_2 \Rightarrow 0$

$$B_3 \Rightarrow \frac{\partial H_y}{\partial x} = (\sigma + j\omega\varepsilon) E_z. \rightarrow (3)$$

(ii) For TE wave

H_x, H_z, E_y exists

$E_x, E_z, H_y = 0$

$$A_1 \Rightarrow \gamma E_y = -\mu j\omega H_x$$

$A_2 \Rightarrow 0$

$$A_3 \Rightarrow \frac{\partial E_y}{\partial x} = -\mu j\omega H_z.$$

$B_1 \Rightarrow 0$

$$B_2 \Rightarrow \frac{\partial H_z}{\partial x} + \gamma H_x = -(\sigma + j\omega\varepsilon) E_y \quad (i)$$

$B_3 \Rightarrow 0$

iii) For TEM wave.

Sub $E_z = 0$ in TM wave:

$$\gamma E_x = \mu j\omega H_y$$

$$\gamma H_y = (\sigma + j\omega\varepsilon) E_x$$

$$\frac{\partial H_y}{\partial x} = 0$$

Transmission of TM wave in parallel plane (ii)

The TM equation wave.

$$\frac{\partial E_z}{\partial x} + \gamma E_x = \mp \mu j\omega H_y \longrightarrow (1)$$

$$\gamma H_y = (\sigma + j\omega\varepsilon) E_x \longrightarrow (2)$$

$$\frac{\partial H_y}{\partial x} = (\sigma + j\omega\varepsilon) E_z \longrightarrow (3)$$

differentiate w.r.t = x.

$$\frac{\partial^2 H_y}{\partial x^2} = (\sigma + j\omega\varepsilon)\left(\frac{\partial E_z}{\partial x}\right)$$

From (1) $\boxed{\frac{\partial E_z}{\partial x} = \mu j\omega H_y - \gamma E_x}$

$$\frac{\partial^2 H_y}{\partial x^2} = (\sigma + j\omega\varepsilon)(\mu j\omega H_y - \gamma E_x)$$

From (2) $\boxed{E_x = \frac{\gamma H_y}{(\sigma + j\omega\varepsilon)}}$

$$\frac{\partial^2 H_y}{\partial x^2} = (\sigma + j\omega\varepsilon)\left(\mu j\omega H_y - \frac{\gamma^2 H_y}{\sigma + j\omega\varepsilon}\right)$$

$$= (\sigma + j\omega\varepsilon)\frac{(\mu j\omega H_y(\sigma + j\omega\varepsilon) - \gamma^2 H_y)}{\sigma + j\omega\varepsilon}$$

$\sigma = 0$

$$\frac{\partial^2 H_y}{\partial x^2} = \mu j^2\omega^2\varepsilon H_y - \gamma^2 H_y$$

$$\frac{\partial^2 H_y}{\partial x^2} = -\omega^2\mu\varepsilon H_y - \gamma^2 H_y$$

$$= -H_y(\omega^2\mu\varepsilon + \gamma^2)$$

$$\frac{\partial^2 H_y}{\partial x^2} + H_y(\omega^2\mu\varepsilon + \gamma^2) = 0.$$

$$h^2 = \omega^2\mu\varepsilon + \gamma^2$$

$$\frac{\partial^2 H_y}{\partial x^2} + h^2 H_y = 0$$

$$\boxed{H_y = B_1 \sinh hx + B_2 \cosh hx}$$

Sub H_y in (2) ($\sigma = 0$) find E_x

$$\gamma(B_1 \sinh hx + B_2 \cosh hx) = j\omega\varepsilon E_x$$

$$\boxed{E_x = \frac{\gamma(B_1 \sinh hx + B_2 \cosh hx)}{j\omega\varepsilon}}$$

Sub H_y in (3) ($\sigma = 0$) find E_z

$$\frac{\partial(B_1 \sinh hx + B_2 \cosh hx)}{\partial x} = j\omega\varepsilon E_z$$

$$hB_1 \cosh hx - B_2 h \sinh hx = j\omega\varepsilon E_z$$

$$E_z = \frac{hB_1 \cosh hx - B_2 h \sinh hx}{j\omega\varepsilon}$$

$$\boxed{E_z = \frac{h(B_1 \cosh hx - B_2 \sinh hx)}{j\omega\varepsilon}}$$

0/25 (i) To find f_c

cut off frequency f_c at $x = a/2$

to make $E_z = 0$

(i) $B_1 = B_2 = 0$

(ii) $\frac{ha}{2} = \frac{m\pi}{2}$

$$h^2 = \omega^2 \mu \varepsilon + \gamma^2$$

$$h = (\omega^2 \mu \varepsilon + \gamma^2)^{1/2}$$

Substitute h in $\frac{ha}{2} = \frac{m\pi}{2}$.

$$(\omega^2 \mu \varepsilon + \gamma^2)^{1/2} a = m\pi$$

$$\omega^2 \mu \varepsilon + \gamma^2 = \left(\frac{m\pi}{a}\right)^2$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 - \omega^2 \mu \varepsilon$$

at $\gamma = 0$, $\omega = \omega_c$.

$$\omega_c^2 \mu \varepsilon = \left(\frac{m\pi}{a}\right)^2$$

$$\omega_c^2 = \left(\frac{m\pi}{a}\right)^2 / \mu \varepsilon$$

$$\omega_c = \frac{m\pi}{a\sqrt{\mu \varepsilon}}$$

$$\omega_c = 2\pi f_c$$

$$2\pi f_c = \frac{m\pi}{a\sqrt{\mu\varepsilon}}$$

$$f_c = \frac{m}{2a\sqrt{\mu\varepsilon}}$$

(ii) To find α and β,

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2\left[1 - \frac{\omega^2\mu\varepsilon}{\left(\frac{m\pi}{a}\right)^2}\right]$$

$$= \left(\frac{m\pi}{a}\right)^2\left[1 - \frac{\omega^2}{\left(\frac{m\pi}{a}\right)^2 \frac{1}{\mu\varepsilon}}\right]$$

$$\gamma = \alpha + j\beta$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2\left[1 - \frac{\omega^2}{\omega_c^2}\right]$$

$$\gamma = \frac{m\pi}{a}\sqrt{1 - \frac{\omega^2}{\omega_c^2}}$$

$$\gamma = \frac{m\pi}{a}\sqrt{1 - \frac{f^2}{f_c^2}}$$

(i) when $f > f_c$

$\gamma \Rightarrow$ imaginary

$$\alpha + j\beta = 0$$

$$\alpha = 0$$

$$\beta = \frac{m\pi}{a}\sqrt{1 - \frac{f^2}{f_c^2}}$$

(ii) when $f < f_c$

$\gamma \Rightarrow$ real

$$\alpha + j\beta = 0$$

$$\beta = 0$$

$$\alpha = \frac{m\pi}{a}\sqrt{1 - \frac{f^2}{f_c^2}}$$

TM wave travelling in z direction & propagation constant varies with time and the is equal to phase constant and the equations are modified as.

$$H_y = (B_1 \sinh x + B_2 \cosh x) e^{-j\beta z} \sin wt$$

$$E_x = \frac{j\beta (B_1 \sinh x + B_2 \cosh x)}{jw\varepsilon} e^{-j\beta z} \sin wt$$

$$E_x = \frac{\beta\left(B_1 \sin hx + B_2 \cos hx\right)}{\omega\varepsilon} e^{-j\beta z} \sin\omega t$$

$$E_z = \frac{h\left(B_1 \cos hx - B_2 \sin hx\right)}{j\omega\varepsilon} e^{-j\beta z} \sin\omega t$$

Transmission of TE wave in parallel plane

TE wave equations are

$$\gamma E_y = -\mu j\omega H_x \rightarrow (1)$$

$$\frac{\partial E_y}{\partial x} = -\mu j\omega H_z \rightarrow (2)$$

$$\frac{\partial H_z}{\partial x} + \gamma H_x = -(\sigma + j\omega\varepsilon) E_y \rightarrow (3)$$

Differentiate (2) w.r.t x.

$$\frac{\partial^2 E_y}{\partial x^2} = -\mu j\omega \frac{\partial H_z}{\partial x}$$

from (3) $\frac{\partial H_z}{\partial x} = -(\sigma + j\omega\varepsilon) E_y - \gamma H_x$

Substitute in above equation

$$\frac{\partial^2 E_y}{\partial x^2} = -\mu j\omega\left[-(\sigma + j\omega\varepsilon)E_y - \gamma H_x\right]$$

$[\sigma = 0]$

$$\frac{\partial^2 E_y}{\partial x^2} = -\mu j\omega\left[-j\omega\varepsilon E_y - \gamma H_z\right]$$

from (1) $H_x = \dfrac{-\gamma E_y}{\mu j\omega}$

$$\frac{\partial^2 E_y}{\partial x^2} = -\mu j\omega\left[-j\omega\varepsilon E_y + \frac{\gamma E_y}{\mu j\omega}\right]$$

$$\frac{\partial^2 E_y}{\partial x^2} = -\mu j\omega\left[\frac{-j^2\omega^2\mu\varepsilon E_y + \gamma^2 E_y}{\mu j\omega}\right]$$

$$\frac{\partial^2 E_y}{\partial x^2} = -E_y\left[\omega^2\mu\varepsilon + \gamma^2\right]$$

$[$Sub $h^2 = \omega^2\mu\varepsilon + \gamma^2]$

$$\frac{\partial^2 E_y}{\partial x^2} = -E_y h^2$$

$$\frac{\partial^2 E_y}{\partial x^2} + E_y h^2 = 0$$

$$E_y = B_3 \sinh x + B_4 \cosh x$$

Sub E_y in (2)

$$\frac{\partial(B_3 \sinh x + B_4 \cosh x)}{\partial x} = -\mu j\omega H_z$$

Sub (E_y) in equation (1)

$$\gamma(B_3 \sin hx + B_4 \cos hx) = -\mu j\omega H_x$$

$$H_x = \frac{-\gamma(B_3 \sin hx + B_4 \cos hx)}{\mu j\omega}.$$

E_y in eqn (2)

$$B_3 h \cos hx - B_4 h \sin hx = -\mu j\omega H_z$$

$$H_z = \frac{-h(B_3 \cos hx - B_4 \sin hx)}{\mu j\omega}.$$

(i) To find f_c:

$$H_z = \frac{-h(B_3 \cos hx - B_4 \sin hx)}{\mu j\omega}.$$

$$x = a/2$$

to make $H_z = 0$

(i) $B_3 = B_4 = 0$

(ii) $\frac{m\pi}{2} = \frac{ha}{2}$

$$h^2 = \omega^2 \mu \varepsilon + \gamma^2$$

$h = \sqrt{\omega^2}$

substitute h in $\frac{ha}{2} = \frac{m\pi}{2}$

$$(\omega^2\mu\varepsilon + \gamma^2)^{1/2} a = m\pi.$$

$$\omega^2\mu\varepsilon + \gamma^2 = \left(\frac{m\pi}{a}\right)^2$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon$$

at $\gamma = 0$, $\omega = \omega_c$

$$\omega_c^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2$$

$$\omega_c^2 = \left(\frac{m\pi}{a}\right)^2 / \mu\varepsilon$$

$$\omega_c = \frac{m\pi}{a\sqrt{\mu\varepsilon}}$$

$$[\omega_c = 2\pi f_c]$$

$$f_c = \frac{m}{2a\sqrt{\mu\varepsilon}}.$$

(ii) To find α and β.

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 - \omega^2\mu\varepsilon$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 \left[1 - \frac{\omega^2\mu\varepsilon}{\left(\frac{m\pi}{a}\right)^2}\right]$$

$$= \left(\frac{m\pi}{a}\right)^2 \left[1 - \frac{\omega^2}{\left(\frac{m\pi}{a}\right)^2 \frac{1}{\mu\varepsilon}}\right]$$

$$\gamma = \alpha + j\beta .$$

$$\gamma^2 = \left(\frac{m\pi}{a}\right)^2 \left[1 - \frac{\omega^2}{\omega_c^2}\right]$$

$$\gamma = \frac{m\pi}{a} \sqrt{1 - \frac{\omega^2}{\omega_c^2}}$$

$$\gamma = \frac{m\pi}{a} \sqrt{1 - \frac{f^2}{f_c^2}}$$

(i) when $f > f_c$

γ is imaginary.

$$\alpha = 0$$

$$\beta = \frac{m\pi}{a} \sqrt{1 - \frac{f^2}{f_c^2}}$$

(ii) when $f < f_c$

γ is real

$$\alpha = \frac{m\pi}{a} \sqrt{1 - \frac{f^2}{f_c^2}}$$

$$\beta = 0$$

TE wave travels in z direction & varies with time and propagation constant is equal to phase constant and the equations are modified as.

$$H_y = (B_3 \sinh x + B_4 \cosh x)\, e^{-j\beta z} \sin\omega t$$

$$E_x = \frac{\beta (B_3 \sinh x + B_4 \cosh x)}{\omega \varepsilon} e^{-j\beta z} \sin\omega t$$

$$E_z = \frac{h (B_3 \cosh x - B_4 \sinh x)}{j\omega\varepsilon} e^{-j\beta z} \sin\omega t$$

20-10-23

Transmission of TEM wave in parallel plane.

It is a special type of TM wave with $E_z = 0$

$$H_y = (B_1 \sinh x + B_2 \cosh x)\, e^{-j\beta z} \sin\omega t$$

$$E_x = \frac{\beta (B_1 \sinh x + B_2 \cosh x)}{\omega\varepsilon} e^{-j\beta z} \sin\omega t$$

$$E_x = \frac{\beta H_y}{\omega \varepsilon}$$

$$h^2 = \gamma^2 + \omega^2 \mu \varepsilon$$

$$h = 0$$

$$\gamma^2 + \omega^2 \mu \varepsilon = 0$$

$$\gamma^2 = -\omega^2 \mu \varepsilon$$

$$\gamma = \sqrt{-\omega^2 \mu \varepsilon}$$

$$\gamma = j\omega\sqrt{\mu\varepsilon}$$

$$\alpha + j\beta = j\omega\sqrt{\mu\varepsilon}$$

$$\beta = \omega\sqrt{\mu\varepsilon}$$

$$E_x = \frac{\beta}{\omega\varepsilon} H_y$$

$$\frac{E_x}{H_y} = \frac{\beta}{\omega\varepsilon} = \frac{\omega\sqrt{\mu\varepsilon}}{\omega\varepsilon}$$

$$= \sqrt{\mu/\varepsilon} = \eta$$

19/10/23 - Waveguide:

There are two waveguide,

1. Rectangular waveguide
2. Circular waveguide.

The waveguide is designed for low cost.

A hollow conducting metallic tube of uniform cross section which is used for propagating EM wave. That is guided along the surface. It is called waveguide.

$$\nabla \times H = (\sigma + j\omega\varepsilon)E$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix} = (\sigma + j\omega\varepsilon)[E_x a_x + E_y a_y + E_z a_z]$$

$$a_x\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right] - a_y\left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right] + a_z\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

$$= (\sigma + j\omega\varepsilon)[E_x a_x + E_y a_y + E_z a_z]$$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = (\sigma + j\omega\varepsilon) E_x \rightarrow (1)$$

$$-\left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right] = (\sigma + j\omega\varepsilon) E_y \rightarrow (2)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \sigma + j\omega\varepsilon E_z \rightarrow (3)$$

$\sigma = 0$

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_x \rightarrow (1)$$

$$-\left[\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right] = j\omega\varepsilon E_y \rightarrow (2)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \rightarrow (3)$$

$$\nabla \times E = -j\omega\mu H.$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = -j\omega\mu H$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_x \rightarrow (4)$$

$$-\left[\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right] = -j\omega\mu H_y$$

$$\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} = j\omega\mu H_y. \rightarrow (5)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \rightarrow (6)$$

$$H_y = H_y^{\circ} e^{\gamma z}$$

$$\frac{\partial H_y}{\partial z} = H_y^{\circ} e^{-\gamma z} \cdot x - \gamma$$

$$\frac{\partial H_y}{\partial z} = -\gamma H_y^{\circ} e^{-\gamma z} = -\gamma H_y$$

$$\frac{\partial H_x}{\partial z} = -\gamma H_x$$

$$\frac{\partial E_x}{\partial x} = -\gamma E_x$$

$$\frac{\partial E_y}{\partial z} = -\gamma E_y.$$

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\varepsilon E_x \rightarrow (7)$$

$$-\left[\frac{\partial H_z}{\partial x} + \gamma H_x\right] = j\omega\varepsilon E_y \rightarrow (8)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z \rightarrow (9)$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \rightarrow (10)$$

$$+ \left[\frac{\partial E_x}{\partial x} + \gamma E_x\right] = j\omega\mu H_y \rightarrow (11)$$

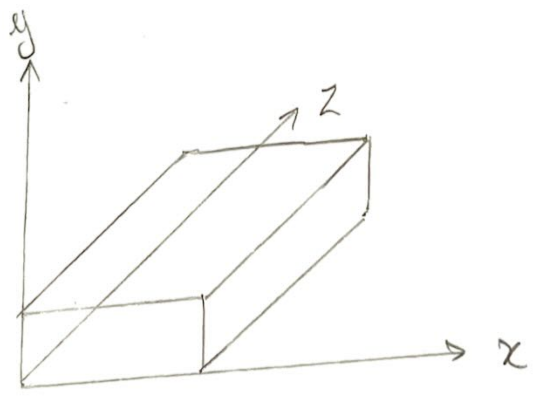
$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \rightarrow (12)$$

In z direction for TE field

$E_z = 0$

In z direction for TM field

$H_z = 0$



From (9)

$$E_x = \frac{1}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial y} + \gamma H_y\right] \rightarrow (13)$$

Sub in (11)

$$\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial y} + \gamma H_y\right] = j\omega\mu H_y$$

$$\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\varepsilon} \cdot \frac{\partial H_z}{\partial y} + \frac{\gamma^2}{j\omega\varepsilon} H_y = j\omega\mu H_y$$

$$\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\varepsilon} \cdot \frac{\partial H_z}{\partial y} = j\omega\mu H_y - \frac{\gamma^2}{j\omega\varepsilon} H_y$$

$$= H_y \left[j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon} \right]$$

$$= H_y \left[\frac{j^2\omega^2\mu\varepsilon - \gamma^2}{j\omega\varepsilon} \right]$$

$$= H_y \left[\frac{-\omega^2\mu\varepsilon - \gamma^2}{j\omega\varepsilon} \right]$$

$$= -H_y \left[\frac{\gamma^2 + \omega^2\mu\varepsilon}{j\omega\varepsilon} \right]$$

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\varepsilon} \cdot \frac{\partial H_z}{\partial y} = -\frac{H_y h^2}{j\omega\varepsilon}$$

$$\boxed{H_y = \frac{-j\omega\varepsilon}{h^2} \left[\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\varepsilon} \frac{\partial H_z}{\partial y} \right] \rightarrow C_1}$$

Substitute in eqn (13)

$$E_x = \frac{1}{j\omega\varepsilon} \left[\frac{\partial H_z}{\partial y} + \frac{-\gamma j\omega\varepsilon}{h^2} \left[\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\varepsilon} \frac{\partial H_z}{\partial y} \right] \right]$$

$$= \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma}{h^2}\left[\frac{\partial E_z}{\partial x} + \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_z}{\partial y}\right]$$

$$= \frac{1}{j\omega\varepsilon}\frac{\partial H_z}{\partial y} - \frac{\gamma \partial E_z}{h^2 \partial x} - \frac{\gamma^2}{j\omega\varepsilon h^2}\cdot\frac{\partial H_z}{\partial y}$$

$$= \frac{\partial H_z}{\partial y}\left[\frac{1}{j\omega\varepsilon} - \frac{\gamma^2}{j\omega\varepsilon h^2}\right] - \frac{\gamma}{h^2}\cdot\frac{\partial E_z}{\partial x}$$

$$= \frac{\partial H_z}{\partial y}\left[\frac{h^2 - \gamma^2}{j\omega\varepsilon h^2}\right] - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}$$

$$h^2 = \gamma^2 + \omega^2\mu\varepsilon$$

$$h^2 - \gamma^2 = \omega^2\mu\varepsilon$$

$$= \frac{\partial H_z}{\partial y}\left[\frac{\omega^2\mu\varepsilon}{j\omega\varepsilon h^2}\right] - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}.$$

$$= \frac{\partial H_z}{\partial y}\left[\frac{\omega\mu}{j h^2}\right] - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x}$$

$$\boxed{E_x = \frac{-j\omega\mu}{h^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial E_z}{\partial x} \rightarrow C_2}$$

From eqn (8)

$$E_y = \frac{-1}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial x} + \gamma H_x\right]$$

Sub in (10)

$$\frac{\partial E_z}{\partial y} + \gamma\left[\frac{-1}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial x} + \gamma H_x\right]\right] = -j\omega\varepsilon H_x$$

$$\frac{\partial E_z}{\partial y} = \frac{-\gamma\partial H_z}{j\omega\varepsilon\partial x} + \frac{-\gamma^2 H_x}{j\omega\varepsilon} = -j\omega\varepsilon H_x$$

$$\frac{\partial E_z}{\partial y} + \frac{\gamma\partial H_z}{j\omega\varepsilon\partial x} = -j\omega\varepsilon H_x + \frac{\gamma^2 H_x}{j\omega\varepsilon}$$

$$\frac{\partial E_z}{\partial y} + \frac{\gamma\partial H_z}{j\omega\varepsilon\partial x} = H_x\left[\frac{\omega^2\varepsilon^2 + \gamma^2 \cancel{H_x}}{j\omega\varepsilon}\right]$$

Substitute eqn (4)

$$\therefore H_x = \frac{-1}{j\omega\mu}\left[\frac{\partial E_z}{\partial y} + \gamma E_y\right]$$

$$= \frac{-1}{j\omega\mu}\frac{\partial E_z}{\partial y} - \frac{\gamma}{j\omega\mu}\times\frac{-1}{j\omega\varepsilon}\left[\frac{\partial H_z}{\partial x} + \gamma H_x\right]$$

$$H_x = -\frac{1}{j\omega\mu}\frac{\partial E_z}{\partial y} + \frac{\gamma}{j^2\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x} + \frac{\gamma^2}{j^2\omega^2\mu\varepsilon}H_x$$

$$H_x + \frac{\gamma^2}{\omega^2\mu\varepsilon}H_x = \frac{-1}{j\omega\mu}\cdot\frac{\partial E_z}{\partial y} - \frac{\gamma}{\omega^2\mu\varepsilon}\cdot\frac{\partial H_z}{\partial x}$$

$$H_x\left[\frac{\omega^2\mu\varepsilon+\gamma^2}{\omega^2\mu\varepsilon}\right]=\frac{-1}{j\omega\mu}\cdot\frac{\partial E_z}{\partial y}\cdot\frac{-\gamma}{\omega^2\mu\varepsilon}\frac{\partial H_z}{\partial x}$$

$$H_x=\frac{j\omega\varepsilon}{h^2}\frac{\partial E_z}{\partial y}-\frac{\gamma}{h^2}\cdot\frac{\partial H_z}{\partial x}.$$

Substitute H_x in E_y

$$E_y=\frac{-1}{j\omega\mu}\left[\frac{\partial E_z}{\partial y}+\gamma E_y\right]$$

$$E_y=\frac{-1}{j\omega\mu}\left[\frac{\partial H_z}{\partial x}\right]-\frac{\gamma}{j\omega\varepsilon}\left[\frac{j\omega\varepsilon}{h^2}\cdot\frac{\partial E_z}{\partial y}-\frac{\gamma}{h^2}\cdot\frac{\partial H_z}{\partial x}\right]$$

$$E_y=-\frac{1}{j\omega\mu}\frac{\partial H_z}{\partial x}-\frac{\gamma}{j\omega\varepsilon}\left[\frac{j\omega\varepsilon}{h^2}\cdot\frac{\partial E_z}{\partial y}+\frac{\gamma^2}{h^2}\cdot\frac{\partial H_z}{\partial x}\right]$$

$$E_y=-\frac{1}{j\omega\mu}\frac{\partial H_z}{\partial x}-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y}\cdot\frac{\gamma^2}{j\omega\varepsilon h^2}\frac{\partial H_z}{\partial x}$$

$$E_y=-\frac{\partial H_z}{\partial x}\left[\frac{1}{j\omega\varepsilon}+\frac{\gamma^2}{j\omega\varepsilon h^2}\right]-\frac{\gamma}{h^2}\frac{\partial E_z}{\partial y}$$

$$E_y=\frac{j\omega\mu}{h^2}\cdot\frac{\partial H_z}{\partial x}-\frac{\gamma}{h^2}\cdot\frac{\partial E_z}{\partial y}.$$

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Transverse electric waves in rectangular waveguide.

For TE wave $E_z = 0$ $H_z \neq 0$

$$E_x = \frac{-j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial y}\right)$$

$$H_x = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

$$E_y = \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

$$H_y = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial y}\right)$$

$$H_z = XYe^{-\gamma z}$$

$$X = C_1 \cos Ax + C_2 \sin Ax$$

$$Y = C_3 \cos By + C_4 \sin By$$

$$H_z = \left(C_1 C_3 \cos Ax \cos By + C_2 C_3 \sin Ax \cos By + C_1 C_4 \cos Ax \sin By + C_2 C_4 \sin Ax \sin By\right)e^{-\gamma z}$$

The Boundary condition.

(i) $E_x = 0$ at $x = 0$ ~~&~~ $x = a$

(ii) $E_x = 0$ at $y = 0$ & $y = b$

$$E_x = \frac{-j\omega\mu}{h^2}\left[\frac{\partial}{\partial y}\left(C_1C_3\cos Ax\cos By + C_1C_4\cos Ax\sin By + C_2C_3\sin Ax\cos By + C_2C_4\sin Ax\sin By\right)e^{-\gamma z}\right]$$

$$E_x = \frac{-j\omega\mu}{h^2}\left[C_1C_3\cos Ax(-\sin By)B + C_1C_4\cos Ax\cos By\,B + C_2C_3\sin Ax(-\sin By)\cdot B + C_2C_4\sin Ax\cos By\cdot B\right]e^{-\gamma z}$$

$\cos 0 = 1$
$\sin 0 = 0$

$$= \frac{-j\omega\mu B}{h^2}\left[C_1C_3\cos Ax(-\sin By) + C_1C_4\cos Ax\cos By + C_2C_3\sin Ax(-\sin By) + C_2C_4\sin Ax\cos By\right]e^{-\gamma z} \rightarrow (1)$$

Substitute

$E_x = 0$ at $x = 0$.

$$0 = \frac{-j\omega\mu B}{h^2}\left[C_1C_3\cancel{\cos}A(-\sin By) + C_1C_4\cancel{\cos}A\cos By\right]e^{-\gamma z}$$

$$C_1C_3\sin By - C_1C_4\cos By = 0$$

$$C_1 = 0$$

Sub $C_1 = 0$ in (1)

$$E_x = \frac{-j\omega\mu B}{h^2}\left[C_2C_3\sin Ax(-\sin By) + C_2C_4\sin Ax\cos By\right]e^{-\gamma z} \rightarrow (2)$$

$$= \frac{j\omega\mu B}{h^2}\left[C_2C_3\sin Ax\sin By - C_2C_4\sin Ax\cos By\right]e^{-\gamma z} \rightarrow (2)$$

$E_x = 0$ at $y = 0$ in (2)

$$0 = \frac{j\omega\mu B}{h^2}\left[-C_2 C_4 \sin Ax\right] e^{-\gamma z}$$

$$C_2 C_4 \sin Ax = 0$$

$$C_4 = 0$$

Substitute $C_4 = 0$ in equation (2).

$$E_x = \frac{j\omega\mu B}{h^2}\left[C_2 C_3 \sin Ax \sin By\right] e^{-\gamma z} \longrightarrow (3)$$

Sub $E_x = 0$ at $x = a$ at (3)

$$0 = \frac{j\omega\mu B}{h^2}\left[C_2 C_3 \sin Aa \sin By\right] e^{-\gamma z}$$

$$\sin Aa = 0$$

$$Aa = \sin^{-1}(0)$$

$$A = \frac{m\pi}{a}$$

Sub in eqn (3).

$$E_x = \frac{j\omega\mu B}{h^2}\left[C_2 C_3 \sin\left(\frac{m\pi}{a}\right)x \sin By\right] e^{-\gamma z} \longrightarrow (4)$$

$E_x = 0$ at $y = b$ in (4)

$$0 = \frac{j\omega\mu B}{h^2}\left[C_2 C_3 \sin\left(\frac{m\pi}{a}\right)x \sin Bb\right]e^{-\gamma z}$$

$$\sin Bb = 0$$

$$B = \frac{n\pi}{b}.$$ substitute in eqn (4)

$$\boxed{E_x = \frac{j\omega\mu B}{h^2}\left[C_2 C_3 \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\right]e^{-\gamma z}}$$

From ~~TE~~ basic equation

$$\cancel{H_x} = \frac{-\gamma}{h^2}\frac{\partial H_z}{\partial x}$$ $[E_z = 0]$

sub $C_1 = 0$, $C_4 = 0$ in H_z equation

$$H_z = \left(C_2 C_3 \sin Ax \cos By\right)e^{-\gamma z}$$

$$E_y = \frac{j\omega\mu}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

$$E_y = \frac{j\omega\mu}{h^2}\left[C_2 C_3 \cos Ax (A) \cos By\right]e^{-\gamma z}.$$

$$\boxed{E_y = \frac{j\omega\mu}{h^2}\left[C_2 C_3 \frac{m\pi}{a}, \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\right] e^{-\gamma z}}$$

$$H_x = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial x}\right)$$

$$H_x = \frac{-\gamma}{h^2}\left(C_2 C_3 A \cos Ax \cos By\right) e^{-\gamma z}.$$

$$\boxed{H_x = \frac{-\gamma}{h^2}\left[C_2 C_3 \frac{m\pi}{a} \cos\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y\right] e^{-\gamma z}.}$$

$$H_y = \frac{-\gamma}{h^2}\left(\frac{\partial H_z}{\partial y}\right)$$

$$= \frac{-\gamma}{h^2}\left[C_2 C_3 \sin Ax\,(-\sin By).B\right] e^{-\gamma z}$$

$$\boxed{H_y = \frac{\gamma}{h^2}\left[C_2 C_3 \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y\right] e^{-\gamma z}}$$

$$h^2 = A^2 + B^2$$

$$= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2.$$

Dominant & Degenerate mode.

The mode which has lowest cutoff frequency and lowest attenuation is called Dominant